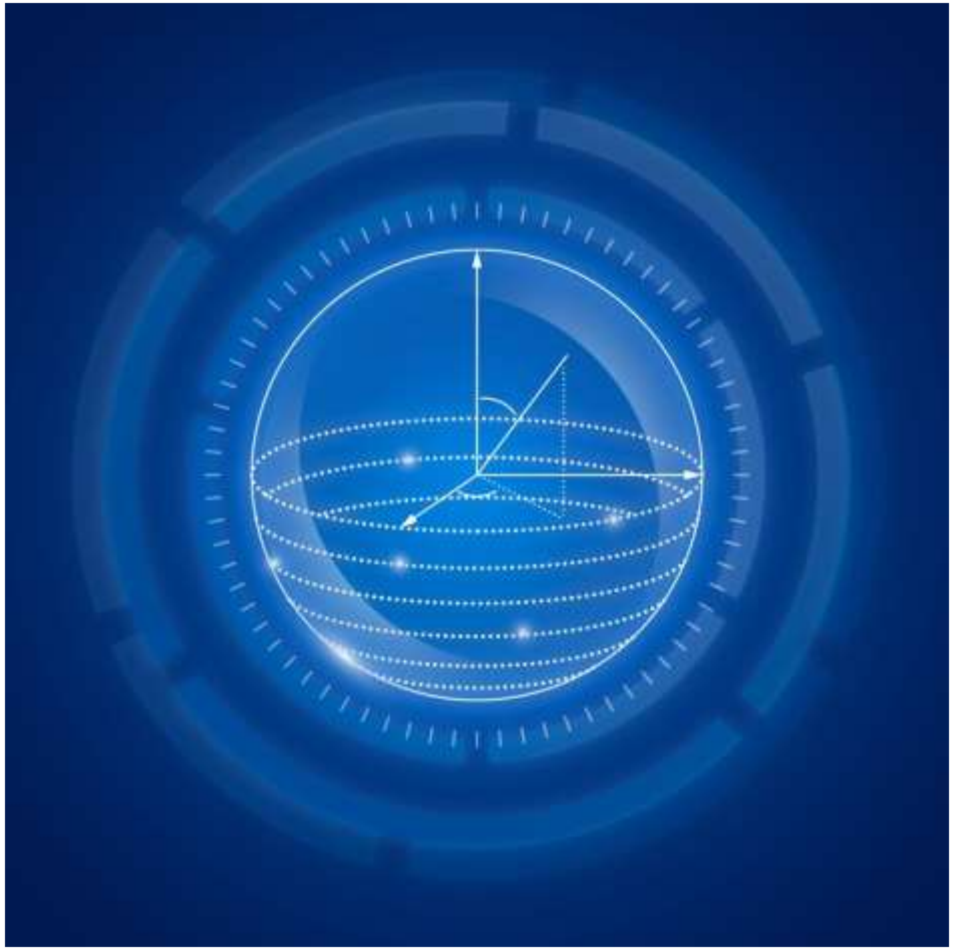


Scalar product and vector product

Vector Product(Cross product)

Definition

$\vec{u}$ and $\vec{v}$ are two vectors in space.

O is any point of the space.

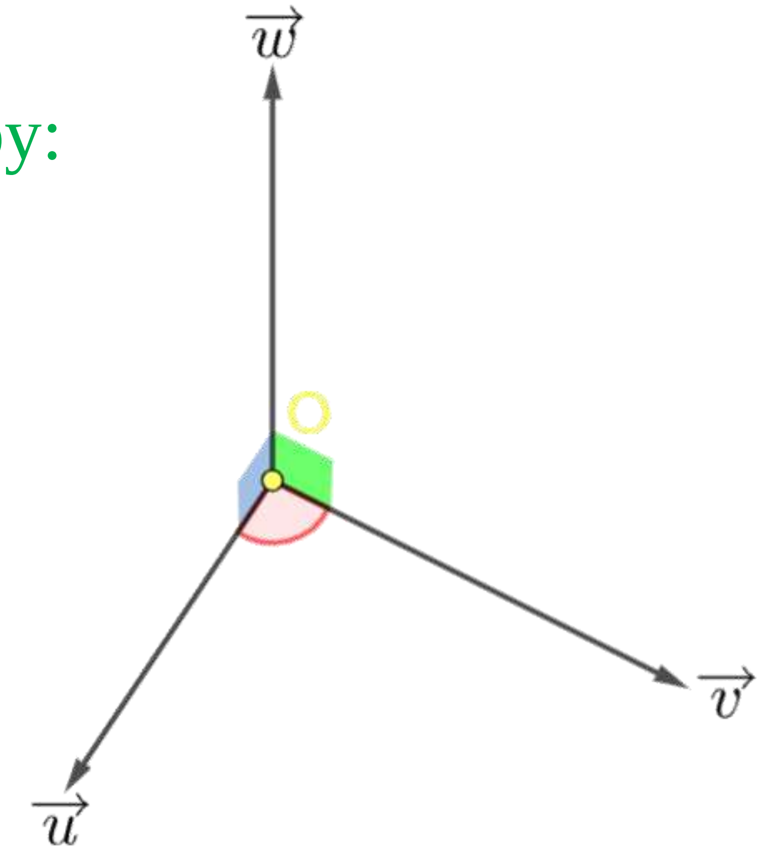
The vector product of two vectors is a vector denoted by:

$$\vec{u} \times \vec{v} \quad \text{or} \quad \vec{u} \wedge \vec{v}$$

❶ $\vec{u}$ and $\vec{v}$ are two non collinear vectors.

$\vec{w} = \vec{u} \wedge \vec{v}$

- ❖ $\vec{w}$ is orthogonal to $\vec{u}$ & $\vec{v}$
- ❖ $\vec{w}$ is orthogonal to the plane containing $\vec{u}$ & $\vec{v}$.
- ❖ The system $(O; \vec{u}; \vec{v}; \vec{w})$ is direct.
- ❖ $||\vec{w}|| = ||\vec{u}|| \times ||\vec{v}|| \times |\sin(\vec{u}; \vec{v})|$



Vector Product(Cross product)

Definition

$\vec{u}$ and $\vec{v}$ are two vectors in space.

O is any point of the space.

The vector product of two vectors is a vector denoted by:

$$\vec{u} \times \vec{v} \quad \text{or} \quad \vec{u} \wedge \vec{v}$$

❷ $\vec{u}$ and $\vec{v}$ are collinear vectors:

$$\vec{w} = \vec{u} \wedge \vec{v} = \vec{0}$$

Remark:

If $\vec{u} \wedge \vec{v} = \vec{0}$, then:

$\vec{u} = \vec{0}$ or $\vec{v} = \vec{0}$

Or

$\vec{u}$ and $\vec{v}$ are collinear vectors

Vector Product(Cross product)

Properties

❶ $\vec{u} \wedge \vec{u} = \vec{0}$

❷ $\vec{u}.(\vec{u} \wedge \vec{v}) = 0$

since $\vec{u} \wedge \vec{v}$ is orthogonal to $\vec{u}$.

❸ $\vec{u} \wedge \vec{v} = -\vec{v} \wedge \vec{u}$

❹ $(a\vec{u}) \wedge \vec{v} = \vec{u} \wedge (a\vec{v}) = a(\vec{u} \wedge \vec{v})$; $a \in \mathbb{R}$

❺ $\vec{u} \wedge \vec{v}$ is maximal when $\vec{u} \perp \vec{v}$

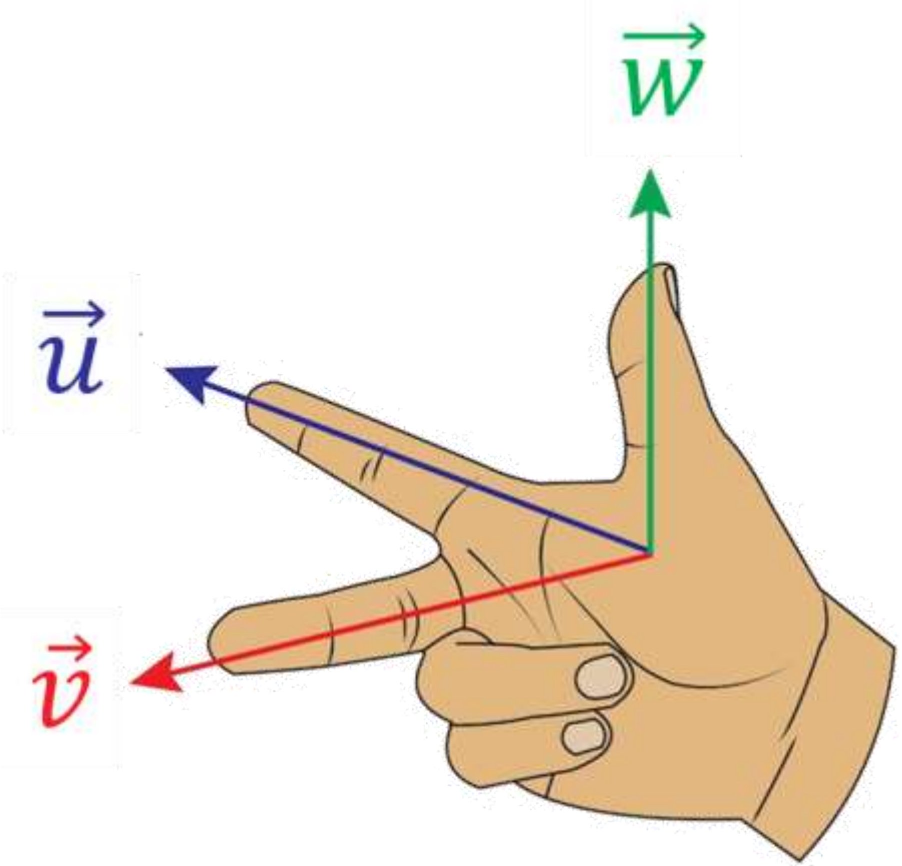
Since $\left||\vec{u} \wedge \vec{v}|\right| = \left||\vec{u}|\right| \times \left||\vec{v}|\right| \times |\sin(\vec{u};\vec{v})|$ and $|\sin(\vec{u};\vec{v})| \leq 1$

When $\vec{u} \perp \vec{v}$, $|\sin(\vec{u};\vec{v})| = 1$

❻ $\vec{u} \wedge (\vec{v} \pm \vec{w}) = \vec{u} \wedge \vec{v} \pm \vec{u} \wedge \vec{w}$

Vector Product(Cross product)

Remark



The direct sense of the vector: $\vec{w} = \vec{u} \wedge \vec{v}$ can be determined using the three fingers of the right hand.

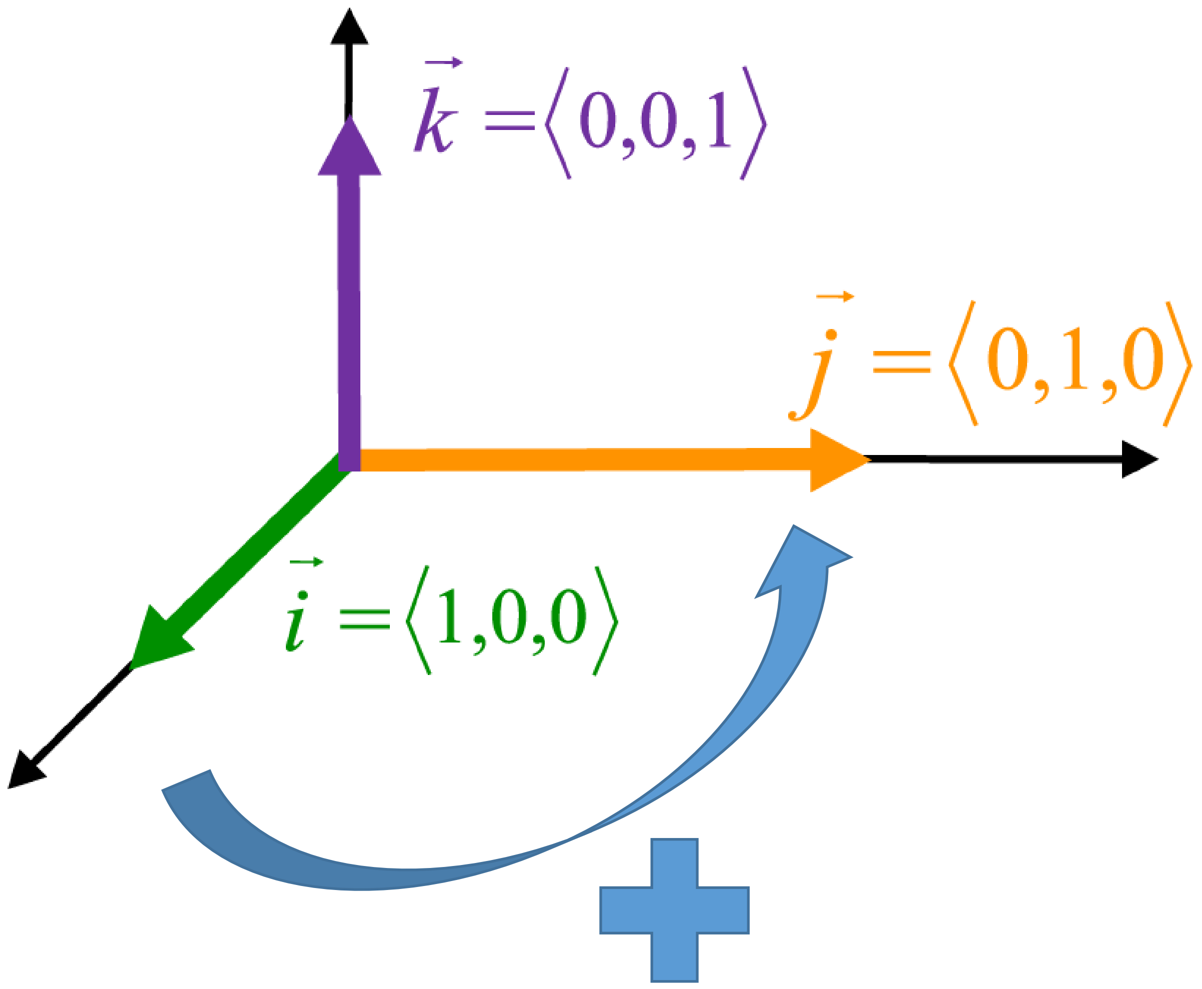
Index: $\vec{u}$

Middle: $\vec{v}$

The direction of the thumb finger determines that of $\vec{w}$.

Vector Product(Cross product)

Remark



$$\vec{i} \wedge \vec{j} = \vec{k}$$
$$\vec{j} \wedge \vec{k} = \vec{i}$$
$$\vec{k} \wedge \vec{i} = \vec{j}$$

Vector Product(Cross product)

Applications of the vector product

❶ Collinearity and parallelism

If $\overrightarrow{AB} \wedge \overrightarrow{AC} = \vec{0}$, then A, B and C are collinear points.

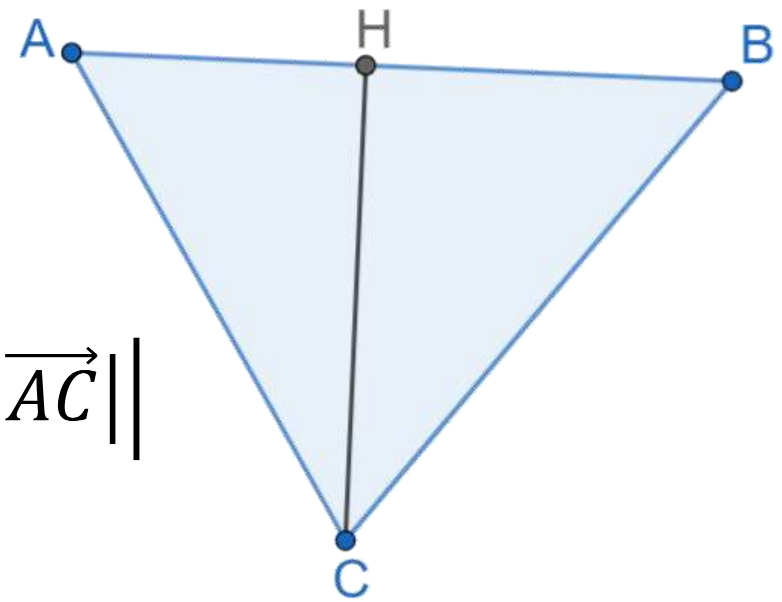
If $\overrightarrow{AB} \wedge \overrightarrow{CD} = \vec{0}$, then (AB)//(CD)

❷ Area of a triangle

$A = \frac{1}{2}\left\|\overrightarrow{AB} \wedge \overrightarrow{AC}\right\|$

Since: $A = \frac{1}{2} CH \times AB$

$= \frac{1}{2} \times AC \sin\hat{A} \times AB = \frac{1}{2}\left\|\overrightarrow{AB} \wedge \overrightarrow{AC}\right\|$



Vector Product(Cross product)

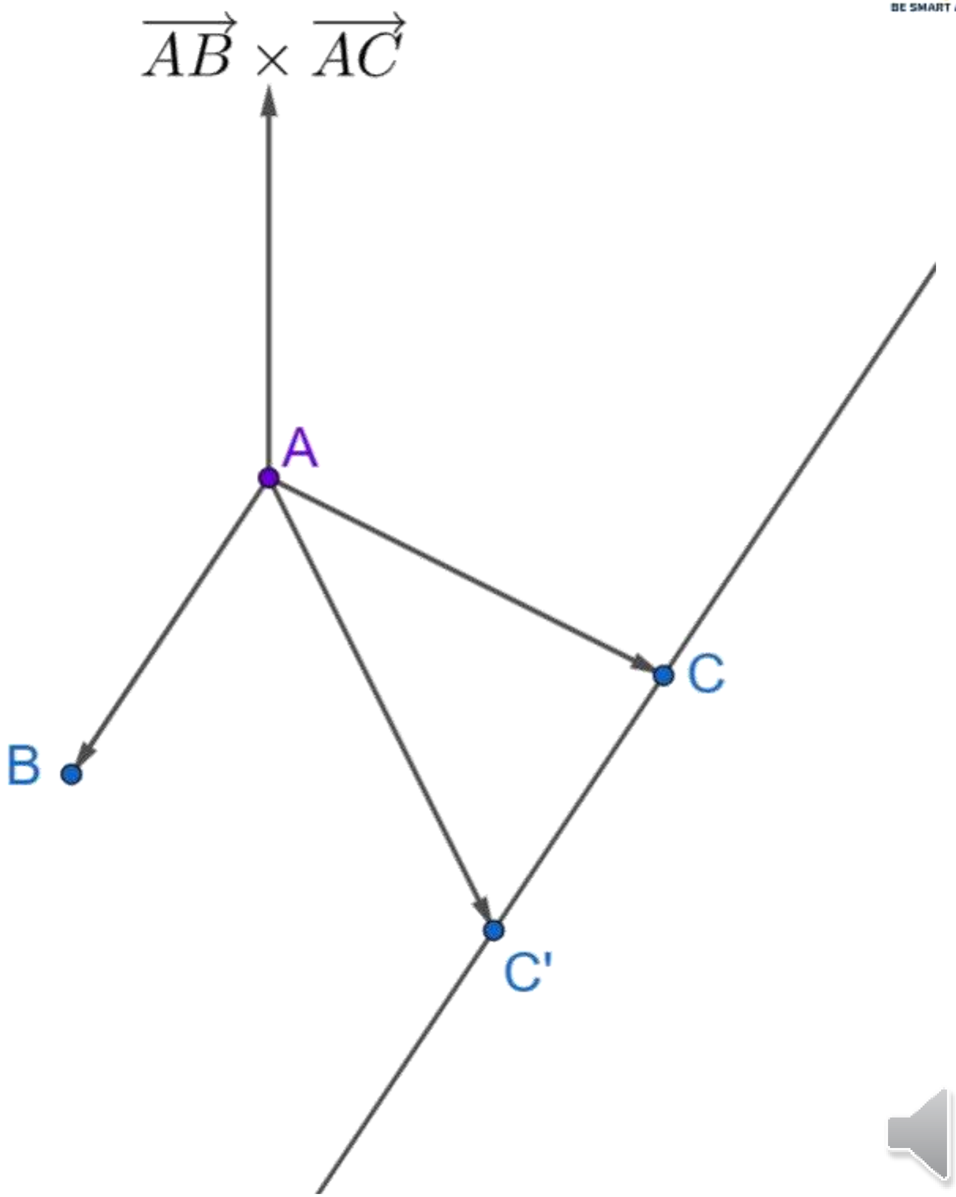
Applications of the vector product

❸ Invariance

If C moves on a line parallel to (AB), then $\overrightarrow{AB} \wedge \overrightarrow{AC}$ does not change.

Since:

$$\overrightarrow{AB} \wedge \overrightarrow{AC} = \overrightarrow{AB} \wedge \left(\overrightarrow{AC'} + \overrightarrow{C'C}\right)$$
$$= \overrightarrow{AB} \wedge \overrightarrow{AC'} + \overrightarrow{AB} \wedge \overrightarrow{C'C}$$
$$= \overrightarrow{AB} \wedge \overrightarrow{AC'} + \vec{0} = \overrightarrow{AB} \wedge \overrightarrow{AC'}$$



Vector Product(Cross product)

Applications of the vector product

❸ Normal vector to a plane

If $\vec{u} = \overrightarrow{AB} \wedge \overrightarrow{AC}$, then $\vec{u}$ is a normal vector to the plane (ABC)